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TECHNICAL REPORT

DEFLECTION OF THE
VERTICAL COMPUTATION
BY ELECTRONIC COMPUTER

Using Areas Bounded by Geographic Coordinates

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Marine Surveys Division

1962



U. S. NAVAL OCEANOGRAPHIC OFFICE
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ABSTRACT

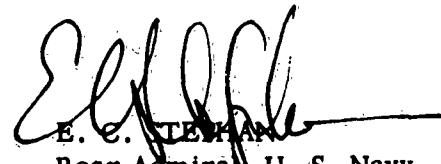
A computational procedure for the gravimetric determination of the deflection of the vertical is presented in this report. This procedure uses the square development exclusively. The effect is computed for the area extending from the computation point to the outermost limits for which data is available.

The mathematical basis for the computations is the Stokes formula for the undulation of the geoid, and the companion formulas of Vening Meinesz for the deflection components. Deflection formulas of Vening Meinesz are given in the appendix.

The deflection components for two areas were determined using an electronic computer and the new procedure. The results of these computations were compared with hand computations which were completed using templates with circles and radial lines.

FOREWORD

» Improved design in gravity measuring equipment has resulted in the accumulation of gravity data in an ever-increasing volume. Technology for processing such data has not kept pace with the continually improving means of data gathering. In the past gravity data has been processed manually, by a technique that has become known as the classic solution, whereby templates consisting of circular rings and radial lines are used. The accuracy of the manual solution has been good, but the method is so laborious that the solution of a great many points would require an excessive amount of time. The computational procedure presented in this report enables the rapid processing of gravity data by an automated technique. The storage requirements of an electronic computer allow and encourage an increase in the accumulation of gravity data. The computational technique ensures that the data will be used for the determination of the deflection components.


E. C. TERHUNE
Rear Admiral U. S. Navy
Commander

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PREFACE

The need for the computation of deflection components by an automated process is universally recognized. The best method by which the deflection components can be obtained is perhaps subject to debate. The method presented in this report attempts to standardize storage requirements and to minimize the need for total number of stored gravity values. Accuracy of the square method of computation appears to equal that of the classical ring procedure. The area immediately surrounding the point is evaluated precisely, rather than approximated. The zone method of development is suited advantageously to solution by electronic computer.

Appreciation is expressed to the following individuals who have contributed to this technical report: Mr. Abraham Balak for his valuable technical advice, Mr. Nathan Fishel for his contribution of comparative computations, Mr. Albert McCahan for his contribution of gravity data, and Mr. A. E. Craig for the encouragement and support necessary to make the solution possible.

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A. INTRODUCTION

When the position of a point on the earth's surface is determined by astronomic observations, the results are referred to the direction of gravity as defined by the leveling device on the observing instrument. Due to the irregular distribution of masses throughout the earth, the level surface of reference varies from place to place in an irregular manner. The geodetic computations are referred to the regular surface of the reference ellipsoid. The deflection of the vertical components indicates the angular tilt between the normal of the ellipsoid and the normal of the level surface.

The determination of the deflection components has always been a time-consuming and tedious undertaking. This report describes a computational method whereby the procedure can be automated. The determination of the effect includes the area from the computation point to the outer limits for which data is available.

For the gravimetric determination of the deflection components, two general methods have been employed. The first and older of the two methods uses templates consisting of circular rings and radial lines. The procedure consists of a graphic determination of the anomaly value for each segment of area. The anomaly value for each radial segment is summed; the summation is then multiplied by a factor which will give the two deflection components. The method is accurate though archaic, since each step has to be performed manually. The method cannot be automated since the anomalies are for area segments that have a unique reference to a given computation point.

The second method employs a technique whereby the anomalies are permanently stored for areas defined as geographic quadrilaterals. This method is referred to as the computation with squares. The greatest advantage is that the anomaly values do not have to be re-estimated for each deflection computation. This technique is well suited for automation, but until now this method has been used only for the determination of the effect of the outer area. The circular method has been used to complete the evaluation of the inner area. The disadvantage here is that the manual circular method must still be used in order to finish the determination of the deflection.

The method presented in this report is for a computational procedure accomplished by the use of squares. The main disadvantage of previous computation by squares has been overcome. A new method for the summation of the anomalies for the inner area now makes it possible to continue the deflection computation right up to the computation point. The advantage is that the whole operation can

be completely automated. The anomalies are permanently stored on magnetic tape, and the computations are performed in one operation on the electronic computer.

A zone system of squares will be developed for the determination of the deflection effect. Every square in a given zone is of the same size; it will be shown that the size of the squares in any given zone are such as to ensure an accurate deflection determination.

The storage of anomalies is a critical consideration for this type of approach. The system imposes storage requirements which must be satisfied in order to implement the new method.

A new method must be compared as to accuracy and economy with the method that is presently being used. The hand computation method which uses the templates will satisfy the check for accuracy and also provide a comparison with the new method. Two problems have been completed by both methods and the results are presented in Section G.

B. STRUCTURE OF SQUARE METHOD

A zone system of squares is developed, whereby 72 squares constitute a zone, and each square in a given zone is of the same size. See Figures 1 and 2. A zone is determined by dividing a given square area into 81 smaller squares; the 9 innermost of these squares are deleted, and the remaining 72 squares constitute the zone. The next zone, approaching the center, is determined by taking the 9 squares which were deleted from the preceding zone and subdividing this area into 81 squares. The 9 innermost of these squares are deleted, and the remaining 72 squares constitute this zone. The same procedure is followed with all additional squares.

Zones are extended inward and outward. Any square in a given zone has the relationship to any square in an adjacent zone such that the sides of the squares have a ratio of 1:3 proceeding outward and 3:1 proceeding inward. Starting with Zone 1 where the 72 squares are one minute squares, and proceeding outward through successive zones, a total of 8 zones will include all of the area of interest, with the exception of the inner area. Starting with Sub-zone 1, which is just inside Zone 1, and proceeding inward through successive sub-zones, an infinite number of sub-zones will be developed.

1	2	3	4	5	6	7	8	9
f	l	k	i	e	i	k	l	f
10	11	12	13	14	15	16	17	18
l	e	j	k	b	k	j	e	l
19	20	21	22	23	24	25	26	27
k	j	d	g	a	g	d	j	k
28	29	30					31	32
i	k	g					g	k
34	35	36					37	38
e	b	a					a	b
40	41	42					43	44
i	k	g					g	k
46	47	48	49	50	51	52	53	54
k	j	d	g	a	g	d	j	k
55	56	57	58	59	60	61	62	63
l	e	j	k	b	k	j	e	l
64	65	66	67	68	69	70	71	72
f	l	k	i	e	i	k	l	f

Figure 1. Zone system of squares.

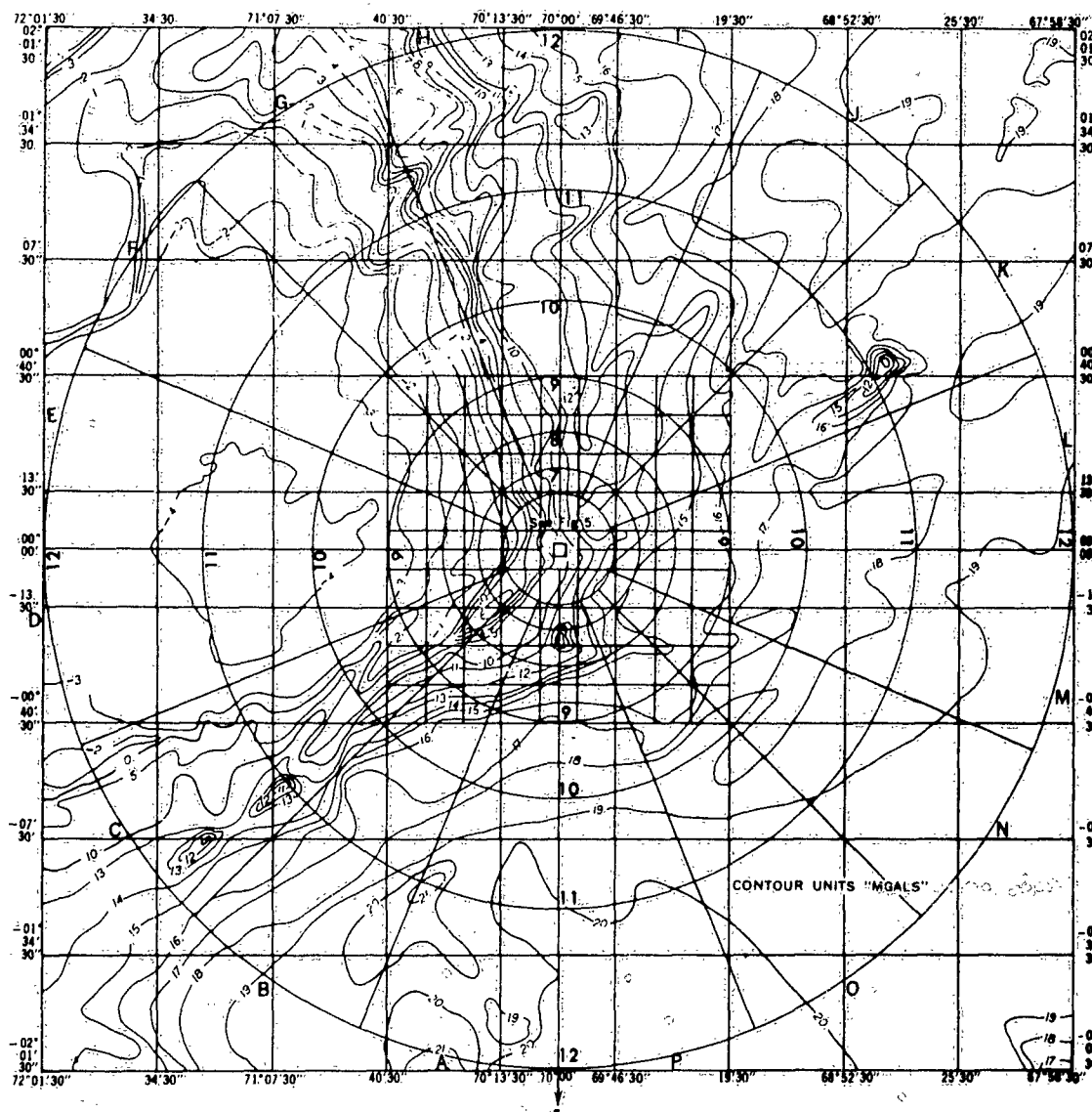


Figure 2. Bathymetric curves used as free-air anomaly contours.

C. CENTER OF GRAVITY OF SQUARES

The squares are actually quadrilaterals with limits defined by geographic coordinates of latitude and longitude. Figures 3a and 3b show that when projected on a plane, a geographic quadrilateral can be represented as an isosceles trapezoid. Obviously, the mean latitude and mean longitude (Figure 3a) will not locate the center of gravity of a quadrilateral.

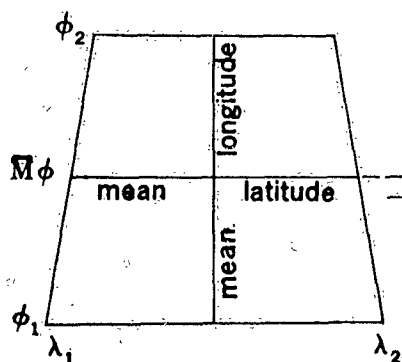


Figure 3a. Mean latitude and mean longitude of a quadrilateral.

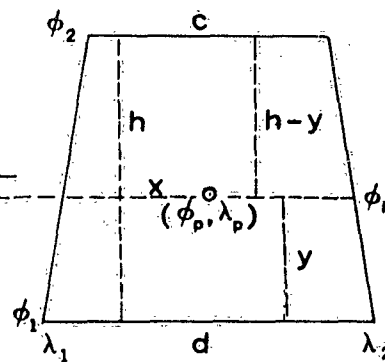


Figure 3b. Center of gravity of a quadrilateral.

In Figure 3b the value x is obtained as follows:

$$x = + \sqrt{\frac{c^2 + d^2}{2}} \quad (3)^*$$

and the value of y is determined by:

$$y = \frac{\text{area of quadrilateral}}{d + x} \quad (4)$$

* See Appendix for expressions (1) and (2)

The expressions (3) and (4) along with the mean longitude define the center of gravity of the quadrilateral.

If the mean latitude and longitude are used as the point to which distance and azimuth are determined from the computation point, a systematic error is introduced into the computations. To eliminate this bias, the center of gravity of each quadrilateral is determined and used. The following tabulation lists the results of several shifts to the center of gravity for quadrilaterals.

	Example 1	Example 2	Example 3
ϕ_1	00° 00'	30° 00'	60° 00'
ϕ_2	20° 00'	50° 00'	80° 00'
$\overline{M} \phi$	10° 00'	40° 00'	70° 00'
ϕ_p	09° 54'	39° 21'	67° 49'
$\overline{M} \phi - \phi_p$	00° 06'	00° 39'	02° 11'

D. COMPUTATION PROCEDURE

For Zones 1 through 8 the computation proceeds zone by zone. Each zone is completely evaluated before going on to the next. The inner area begins with Sub-zone 1 and continues for an infinite number of sub-zones approaching the computation point. All of the sub-zones of the inner area are evaluated at one time by the summation of an infinite series.

Figure 1 shows that the 72 squares that constitute a zone have 12 different geometric relationships with respect to the computation point. These 12 relationships are indicated by letters a through l. Table 1 is given as a proof to show that regardless of sub-zone, the same relative square in each sub-zone exerts a constant deflection force upon the computation point.

For the computation of the inner sub-zones a constant gradient throughout the area is assumed; this condition is more nearly true for each successive sub-zone approaching the computation point. All 72 squares, for all sub-zones, approach the central point in precisely the same mathematical manner. Each square approaches the point at a ratio of two-thirds with respect to the preceding square. By knowing only the first and last term of the series, the summation of the anomalies is as follows:

$$\sum_{i=1}^{\infty} \Delta g_i = \Delta g_1 + \Delta g_2 + \Delta g_3 + \dots + \Delta g_p \quad (5)$$

where successive terms of Δg are determined by the previous and last term as follows:

$$\Delta g_n = 2/3 (\Delta g_p - \Delta g_{n-1}) + \Delta g_{n-1} \quad (6)$$

TABLE 1

	SUB- ZONE 1	SUB- ZONE 2	SUB- ZONE 3		SUB- ZONE N
SIDE	1	$(\frac{1}{3})^1$	$(\frac{1}{3})^2$	$(\frac{1}{3})^{N-1}$
AREA	1	$(\frac{1}{3})^2$	$(\frac{1}{3})^4$	$(\frac{1}{3})^{2(N-1)}$
Ψ_a	2	$2(\frac{1}{3})^1$	$2(\frac{1}{3})^2$	$2(\frac{1}{3})^{N-1}$
F_a	$\frac{1}{4} \gamma M$	=	=	=	$A/\psi_a^2 \gamma M$
Ψ_b	3	$3(\frac{1}{3})^1$	$3(\frac{1}{3})^2$	$3(\frac{1}{3})^{N-1}$
F_b	$\frac{1}{9} \gamma M$	=	=	=	$A/\psi_b^2 \gamma M$
Ψ_c	4	$4(\frac{1}{3})^1$	$4(\frac{1}{3})^2$	$4(\frac{1}{3})^{N-1}$
F_c	$\frac{1}{16} \gamma M$	=	=	=	$A/\psi_c^2 \gamma M$
Ψ_d	$2\sqrt{2}$	$\frac{2}{3^1}\sqrt{2}$	$\frac{2}{3^2}\sqrt{2}$	$\frac{2}{3^{N-1}}\sqrt{2}$
F_d	$\frac{1}{8} \gamma M$	=	=	=	$A/\psi_d^2 \gamma M$
Ψ_e	$3\sqrt{2}$	$\frac{3}{3^1}\sqrt{2}$	$\frac{3}{3^2}\sqrt{2}$	$\frac{3}{3^{N-1}}\sqrt{2}$
F_e	$\frac{1}{18} \gamma M$	=	=	=	$A/\psi_e^2 \gamma M$
Ψ_f	$4\sqrt{2}$	$\frac{4}{3^1}\sqrt{2}$	$\frac{4}{3^2}\sqrt{2}$	$\frac{4}{3^{N-1}}\sqrt{2}$
F_f	$\frac{1}{32} \gamma M$	=	=	=	$A/\psi_f^2 \gamma M$
Ψ_g	$\sqrt{2^2+1^2}$	$\frac{1}{3^1}\sqrt{2^2+1^2}$	$\frac{1}{3^2}\sqrt{2^2+1^2}$	$\frac{1}{3^{N-1}}\sqrt{2^2+1^2}$
F_g	$\frac{1}{9} \gamma M$	=	=	=	$A/\psi_g^2 \gamma M$
Ψ_h	$\sqrt{3^2+1^2}$	$\frac{1}{3^1}\sqrt{3^2+1^2}$	$\frac{1}{3^2}\sqrt{3^2+1^2}$	$\frac{1}{3^{N-1}}\sqrt{3^2+1^2}$
F_h	$\frac{1}{10} \gamma M$	=	=	=	$A/\psi_h^2 \gamma M$
Ψ_i	$\sqrt{4^2+1^2}$	$\frac{1}{3^1}\sqrt{4^2+1^2}$	$\frac{1}{3^2}\sqrt{4^2+1^2}$	$\frac{1}{3^{N-1}}\sqrt{4^2+1^2}$
F_i	$\frac{1}{17} \gamma M$	=	=	=	$A/\psi_i^2 \gamma M$
Ψ_j	$\sqrt{2^2+3^2}$	$\frac{1}{3^1}\sqrt{2^2+3^2}$	$\frac{1}{3^2}\sqrt{2^2+3^2}$	$\frac{1}{3^{N-1}}\sqrt{2^2+3^2}$
F_j	$\frac{1}{13} \gamma M$	=	=	=	$A/\psi_j^2 \gamma M$
Ψ_k	$\sqrt{2^2+4^2}$	$\frac{1}{3^1}\sqrt{2^2+4^2}$	$\frac{1}{3^2}\sqrt{2^2+4^2}$	$\frac{1}{3^{N-1}}\sqrt{2^2+4^2}$
F_k	$\frac{1}{20} \gamma M$	=	=	=	$A/\psi_k^2 \gamma M$
Ψ_l	$\sqrt{3^2+4^2}$	$\frac{1}{3^1}\sqrt{3^2+4^2}$	$\frac{1}{3^2}\sqrt{3^2+4^2}$	$\frac{1}{3^{N-1}}\sqrt{3^2+4^2}$
F_l	$\frac{1}{19} \gamma M$	=	=	=	$A/\psi_l^2 \gamma M$

It can be seen from (5) that if Δg_p is anything other than zero, then the series diverges, and the summation is infinite. Since the gradient rather than the anomaly values creates the deflection effect, the whole anomaly pattern of the inner area can be shifted so that Δg_p is zero. This can always be accomplished, and the summation can always be rendered finite. The following expression achieves the shift and summation simultaneously.

$$\sum_{i=1}^{\infty} \Delta g_i = \frac{(\Delta g_1 - \Delta g_p)(1 - 0.333^{\infty})}{1 - 0.333} \quad (7)$$

The expression (7) reduces to (8):

$$\sum_{i=1}^{\infty} \Delta g_i = 1.5 (\Delta g_1 - \Delta g_p) \quad (8)$$

where Δg_1 is the anomaly for any one of the 72 squares of Sub-zone 1, and Δg_p is the anomaly for the computation point.

With the summation of the inner area achieved by (8), it is only necessary to multiply each summation by the constant effect.

Table 1 was developed for a plane; when a quadrilateral is used, there will be 72 constant geometric relationships rather than the 12 shown in Table 1. This does not impair the accuracy nor restrict the electronic computer. The effect for the constant geometric relationship was tested from the equator to latitude 85°, and the deviation due to the convergency of the meridians amounts to less than 1/100 of one percent.

E. EVALUATION OF DEFLECTION EFFECT OF A SQUARE ON A POINT

The area shown in Figure 4 portrays a rapid change of gradient. If the size of the square UVWX is very large with respect to its distance from the computation point, then the deflection determination for the square will be in error by an excessive amount. If the size of the square UVWX is very small with respect to its distance from the computation point, then the rapid change of gradient will be insignificant, and the deflection determination will be correct. The critical consideration here is the size of a square with respect to its distance from the point. If the squares selected are too large, there will be inaccuracies. If the squares selected are too small, there will be too many of them, and work will be increased. It is the purpose of this section to show that the size of the squares used are such that accuracy is not sacrificed.

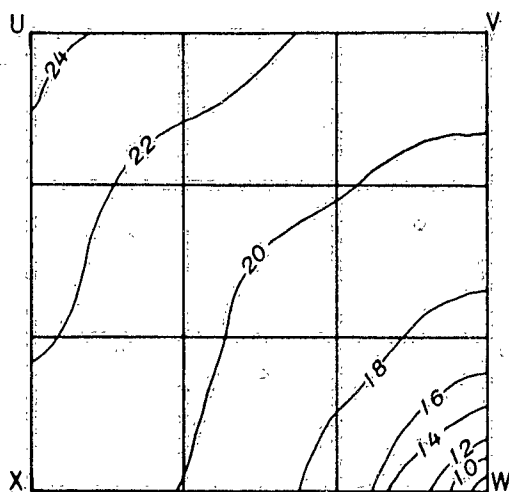


Figure 4. Free-air anomaly contours showing a rapid change of gradient.

Assume that the whole area of square UVWX (Figure 4) coincides with square 52d (Figure 1), and that this area is evaluated directly as square 52d. Square 52d is then divided into 9 smaller squares and each of these is evaluated independently and the 9 deflection effects are then summed. Square UVWX is then successively shifted to squares 48d, 21d, and 25d. In each of these locations (52, 48, 21, and 25), square "d" represents the same geometric relationship with respect to the computation point, and in each of these locations, square UVWX retains the same orientation with U always located at the upper left-hand corner of the square. The same computational procedure is repeated to square UVWX in each of these four locations.

<u>Square</u>	<u>52d</u>	<u>48d</u>	<u>21d</u>	<u>25d</u>
Square UVWX	0".060	0".060	0".060	0".060
9 Squares Σ	<u>0".062</u>	<u>0".061</u>	<u>0".059</u>	<u>0".060</u>
Δ	0".002	0".001	0".001	0".000

The effect of square UVWX is the same in each case, since a mean anomaly for the whole square was used, and the distance and first quadrant azimuth angle remain the same. The effect determined by the 9 smaller squares will change, and the result is a more precise determination. However, the largest difference for Δ is only 0".002, and it should be noted that this difference is found in an area where the gradient changes rapidly.

If Δ in the above comparisons had been large, it would indicate that the square "d" was too large with respect to its distance from the computation point. This indicates from the comparisons that square "d" is of a reasonable size and that good results can be obtained even when a rapid change of gradient occurs within the square. Every square in every zone, in the method presented in this report, is of the same relative size with respect to the computation point. The ratio of square size and distance to the central point is a constant. Therefore, the comparisons given would be valid for any zone.

F. STORAGE OF ANOMALIES

In order to implement the method presented in this report, a library of stored anomalies must be established which is compatible with the computational procedure. It is envisioned that the deflection computations will be for either the center of one-minute squares or the corners of one-minute squares. The following storage requirements will permit the deflection computation for every one-minute square in a one-degree block.

<u>Zone</u>	<u>Area</u>	<u>Storage Square</u>	<u>No. of Squares</u>
8	180° x 360°	5° x 5°	2592
6, 7	61° x 61°	1° x 1°	3721
5	23.6° x 23.6°	10' x 10'	20164
4	10° x 10°	5' x 5'	14400
1, 2, 3	2.5° x 2.5°	1' x 1'	22500
			Σ 63377

The total number of squares required for an initial effort is 63377, but as the library of stored anomalies is increased, the requirements are more easily satisfied. For example, consider a one-degree square adjacent to another one-degree square for which computations have already been performed. The additional number of squares required for this method is 11250, which is a considerable economy over the initial requirements.

G. COMPARISON OF MACHINE SOLUTION WITH CIRCULAR TEMPLATE HAND COMPUTATION

Deflection computations for two specific problems were computed and compared. The first problem was a hypothetical one using bathymetric curves as milligals. The second problem was for the deflection effect at Columbus, Ohio. The gravity data used was a free air anomaly chart that had been prepared at Ohio State University.

Each of the two problems was computed by two methods - the new square method and the circular template method. The square method was performed by an electronic computer (the Bendix G15-D), and the circular method was performed by hand computation. The effect of the outer corners in the square method was deleted, so that the comparisons could be made between areas common to both methods.

In the hypothetical problem (see Figures 2 and 5), the outer radius for the circular method is 2°025. In the Columbus problem, the outer radius was selected as 1°407 since this was the largest radius for which anomaly values were available.

Each of the computations by the circular method was performed twice, by two individuals, to reduce the possibility of error in the comparisons. The results of the comparisons are given below.

Hypothetical Problem

Method Used	ξ	η
Circular	0°345	3°225
Square	0°321	3°185
	Δ 0°024	Δ 0°040

Columbus Problem

Method Used	ξ	η
Circular	-0°677	-5°605
Square	-0°718	-5°397
	Δ -0°041	Δ -0°208

The comparisons between the automated square method of computation and the hand-computed circular method indicate that accuracy requirements have been met. It is impossible to determine which method is more accurate, since differences of the same order as those shown will occur when two individuals using the same method solve a given problem.

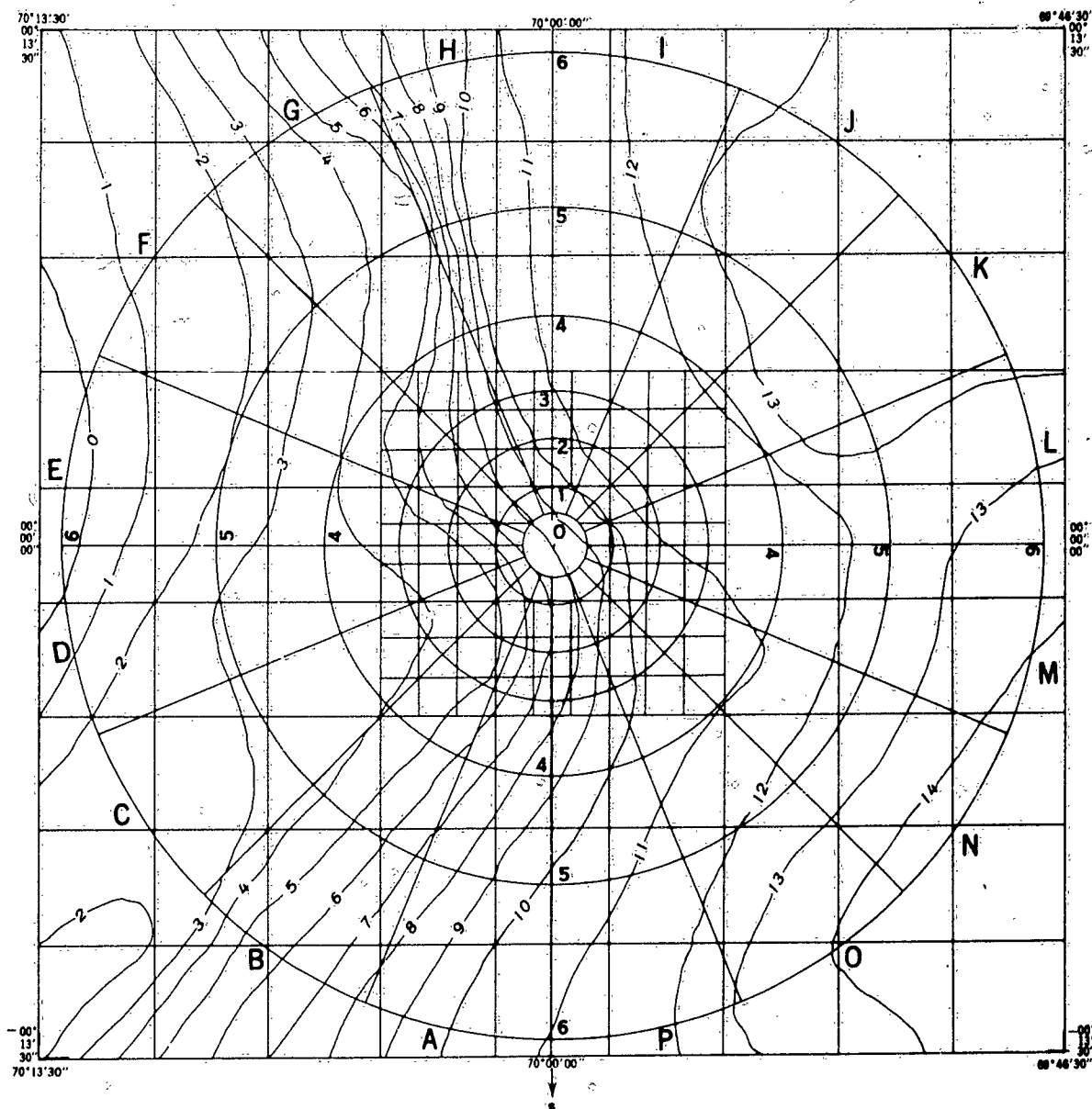


Figure 5. (Insert of Figure 2) Bathymetric curves used as free air anomaly contours.

H. SUMMARY

Number of Computations. The machine solution procedure is not suited advantageously to solving single deflection computations. However, machine solution procedures are highly desirable when large numbers of deflections are necessary.

Accuracy of Data. A comparison of the circular and square computation methods indicates that the results of either method are of the same order of accuracy. These results are within the limits of absolute accuracy. Absolute accuracy for the deflection determination is restricted to plus or minus one second of arc. The principle reason for this restriction is that large areas of the earth's surface remain unsurveyed. Figure 5 shows that the circular-square results are comparable because the individual wedge and square-shaped segments are approximately the same size. Greater accuracy with either method is limited by the small amount of complete gravity coverage.

Use for Data. Availability of large amounts of data would permit publication of tables and graphic contour charts for each of the deflection components.

I. CONCLUSIONS

The time and labor involved in computing deflection components by hand has in the past greatly restricted the application of these corrections in geodetic problems. In the case of triangulation alone, the deflection components can be used to correct the observed horizontal angles. The corrections to the horizontal angles, which can be as large as several seconds, are seldom applied because of the labor involved. When a machine solution for the deflection determination is available, many refinements can be made in many different kinds of work.

The machine solution necessitates storage requirements which are reasonable, and which must be met before the automated method can be achieved. The constantly increasing volume of gravity data along with the geodetic requirements for navigational and geodetic satellites makes it most urgent that the computation be automated.

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APPENDIX I

F. A. Vening Meinesz's Formulas for the Deflection Components

$$\xi'' = -\frac{\text{csc } 1''}{2\pi G} \int_0^\pi \int_0^{2\pi} \Delta g \frac{df(\psi)}{d\psi} \sin \psi \cos a \, d\psi \, da \quad (1)$$

$$\eta'' = -\frac{\text{csc } 1''}{2\pi G} \int_0^\pi \int_0^{2\pi} \Delta g \frac{df(\psi)}{d\psi} \sin \psi \sin a \, d\psi \, da \quad (2)$$

where ξ is the deflection component in the meridian

η is the deflection component in the prime vertical

G is the mean value of gravity in milligals equal to 979.8

Δg is the anomaly in milligals

ψ is the angular distance from the computation point

a is the geodetic azimuth from the south.

APPENDIX II

FORMULAS FOR THE DETERMINATION OF THE UNDULATION OF THE GEOID

The following formulas are for the determination of the undulation of the geoid. When the gradient is a constant in the area about the computation point, the undulation effect for this central area can be determined with formula (15).

The Stoke's Formula for geoid undulation is

$$N = \frac{R}{4\pi G} \int_0^\pi \int_0^{2\pi} \Delta g S(\psi) d\sigma \quad (9)$$

where

$$S(\psi) = 1 + \csc \frac{\psi}{2} - 6 \sin \frac{\psi}{2} - 5 \cos \psi - 3 \cos \psi \log_e \left(\sin \frac{\psi}{2} + \sin^2 \frac{\psi}{2} \right) \quad (10)$$

For a given compartment, N is

$$N(\phi, \lambda) = \frac{R}{4\pi G} \Delta g(\phi, \lambda) S(\psi) d\sigma \quad (11)$$

when $\psi < 1^\circ$, (12) is less than 5% in error.

$$S(\psi) \cong \frac{2}{\psi} \quad (12)$$

Letting

$$C = \left(\frac{R}{4\pi G} \right) \left(\frac{2}{\psi} \right) d\sigma \quad (13)$$

N is

$$N(\phi, \lambda) \cong C \Delta g(\phi, \lambda) \quad (14)$$

The summation for a given numbered square, for successive sub-zones approaching the computation point, may be achieved with (15).

$$\sum_{i=1}^n N_i \cong \left[\Delta g_1 C + \Delta g_2 C \left(\frac{1}{3} \right) + \dots + \Delta g_n C \left(\frac{1}{3} \right)^{n-1} \right] \quad (15a)$$

In closed form, (15a) becomes

$$\sum_{i=1}^n N_i \cong C \left[\Delta g_1 + \frac{1}{9} (\Delta g_1 + 2\Delta g_p) + \frac{\Delta g_p}{6} \right] \quad (15b)$$

The following substitution reduces the error to less than 1%.

$$\sum_{i=1}^n N_i \cong \left[\frac{R}{4\pi G} S(\psi) d\sigma \right] \left[\Delta g_1 + \frac{1}{9} (\Delta g_1 + 2\Delta g_p) + \frac{\Delta g_p}{6} \right] \quad (15c)$$

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